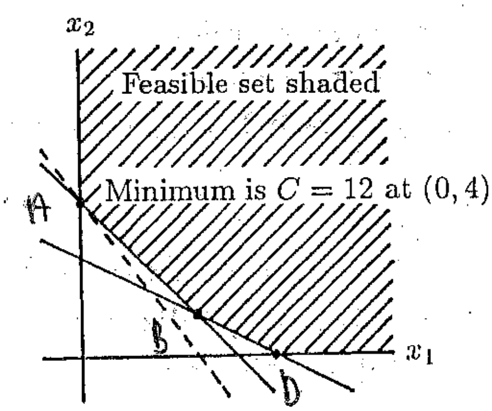
CHAPTER 2 Q + A

SOLVE EACH OF THE FOLLOWING LP PROBLEMS GEOMETRICALLY USING THE METHOD OF CHAPTER 2

(1)
$$\min C = 4x_1 + 3x_2$$
$$x_1 + 2x_2 \geq 5$$
$$x_1 + x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

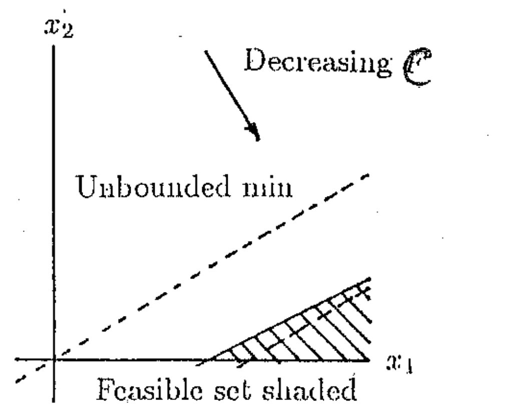
soln



	C
A = (0,4)	12
B = (3,1)	15
D = (5,0)	20

(2)
$$\min C = -3x_1 + 5x_2$$
$$x_1 - 2x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

soln



LP (2) Q+A . F. REGION SHADED

(3) SOLVE

$$\max P = 3x_1 + 2x_2$$
$$x_1 + 2x_2 \le 6$$
$$x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0$$

Soln

(1) $x_1 + 2x_2 = 6$

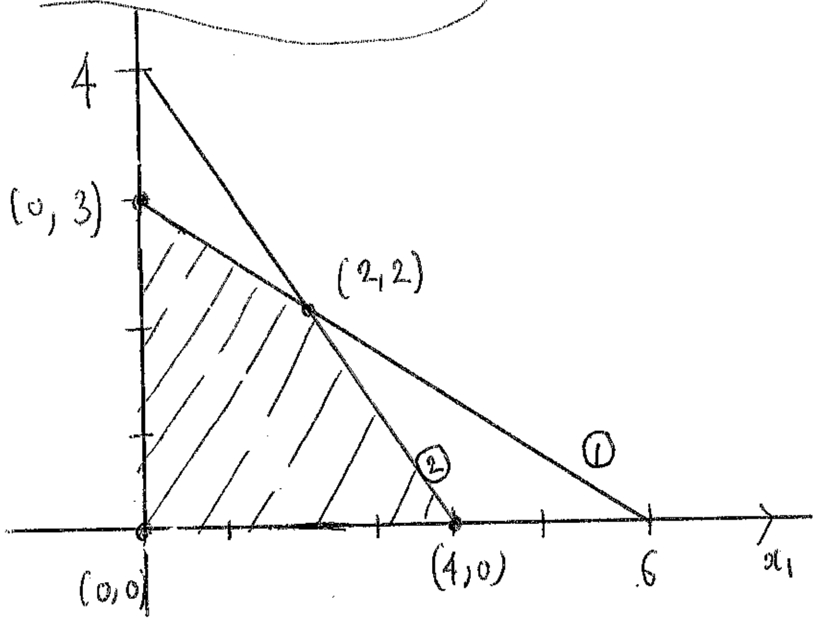
(0, 3)

(6, 0)

(2) $x_1 + x_2 = 4$

(0, 4)

(4, 0)



P max @ (4, 0)

P = 12

CORNER POINT (VERTEX)	$P = 3x_1 + 2x_2$
(0,0)	0
(0,3)	6
(4, 0)	12
(2,2)	10

LP ② F. REGION SHADED

(4) min $C = 4x_1 + x_2$

$2x_1 + 3x_2 \geq 4$

$2x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

① $2x_1 + 3x_2 = 4$

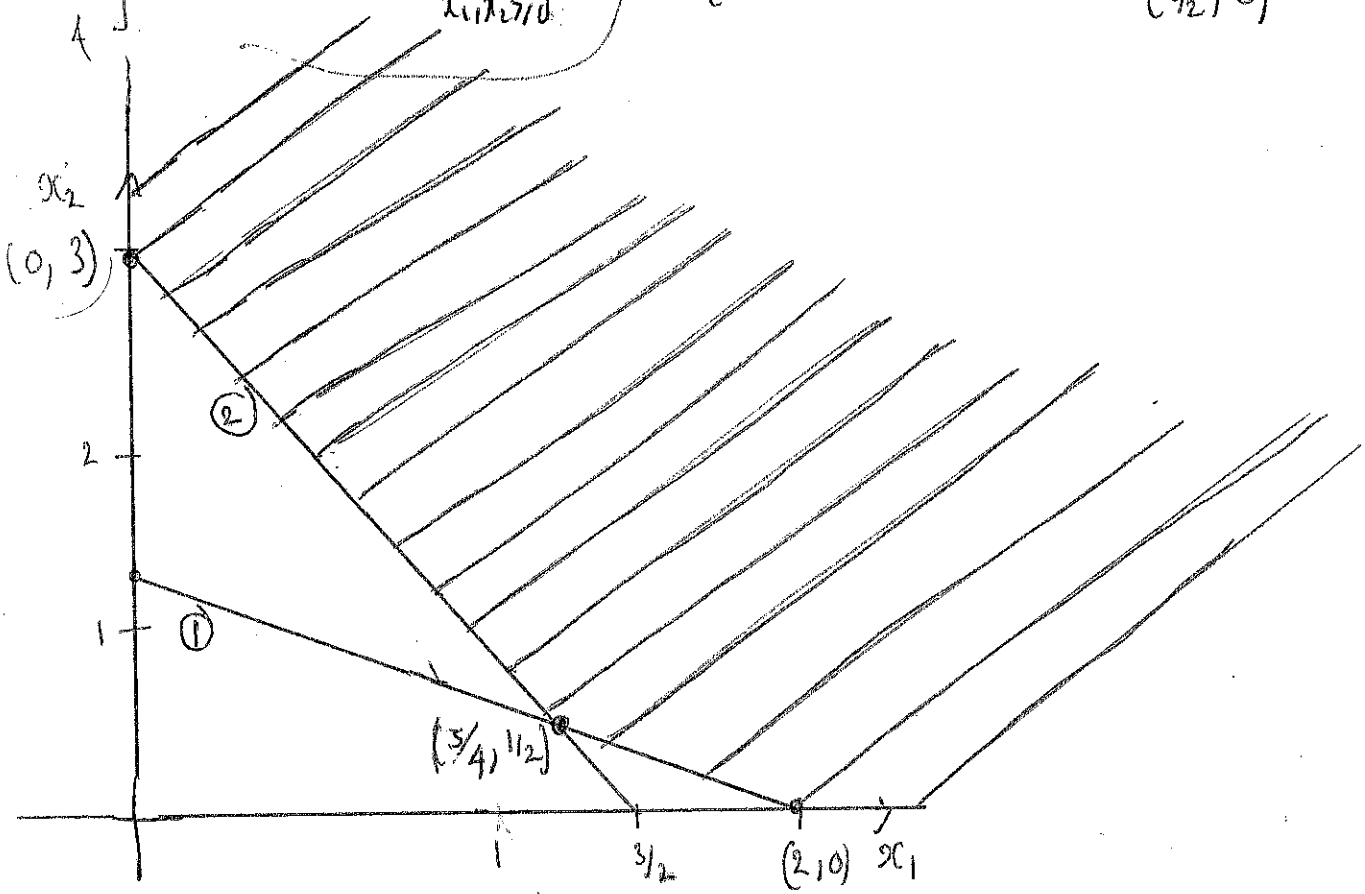
$(0, 4/3)$

$(2, 0)$

② $2x_1 + x_2 = 3$

$(0, 3)$

$(3/2, 0)$



VERTEX	$C = 4x_1 + x_2$
(0, 3)	3
(3/4, 1/2)	5 1/2
(2, 0)	8

$\Rightarrow$ C min @ (0, 3)

C = 3

LP ② F. REGION SHADED

5) $\max P = 3x_1 + 2x_2$

$x_1 - x_2 \geq 5$

$x_1 + x_2 \leq 6$

$x_1, x_2 \geq 0$

① $x_1 - x_2 = 5$

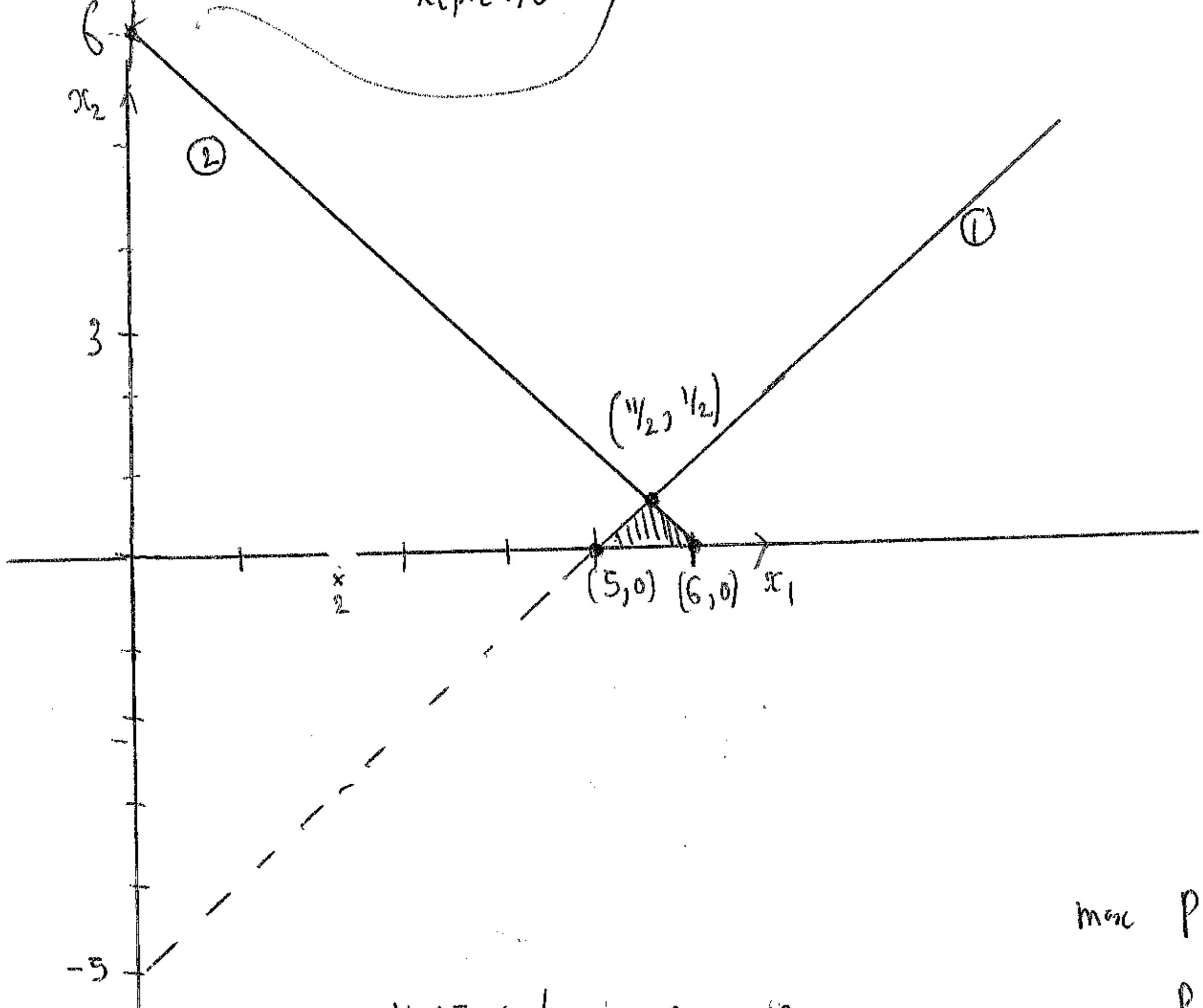
(0, −5)

(5, 0)

② $x_1 + x_2 = 6$

(0, 6)

(6, 0)



VERTEX	$P = 3x_1 + 2x_2$
(11/2, 1/2)	17 1/2
(5, 0)	15
(6, 0)	18

max P @ (6,0)

P = 18

LP ②   F. REGION SHADED

⑥ $\min C = 4x_1 + 3x_2$

$x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \geq 8$

$x_1, x_2 \geq 0$

① $x_1 + x_2 = 2$

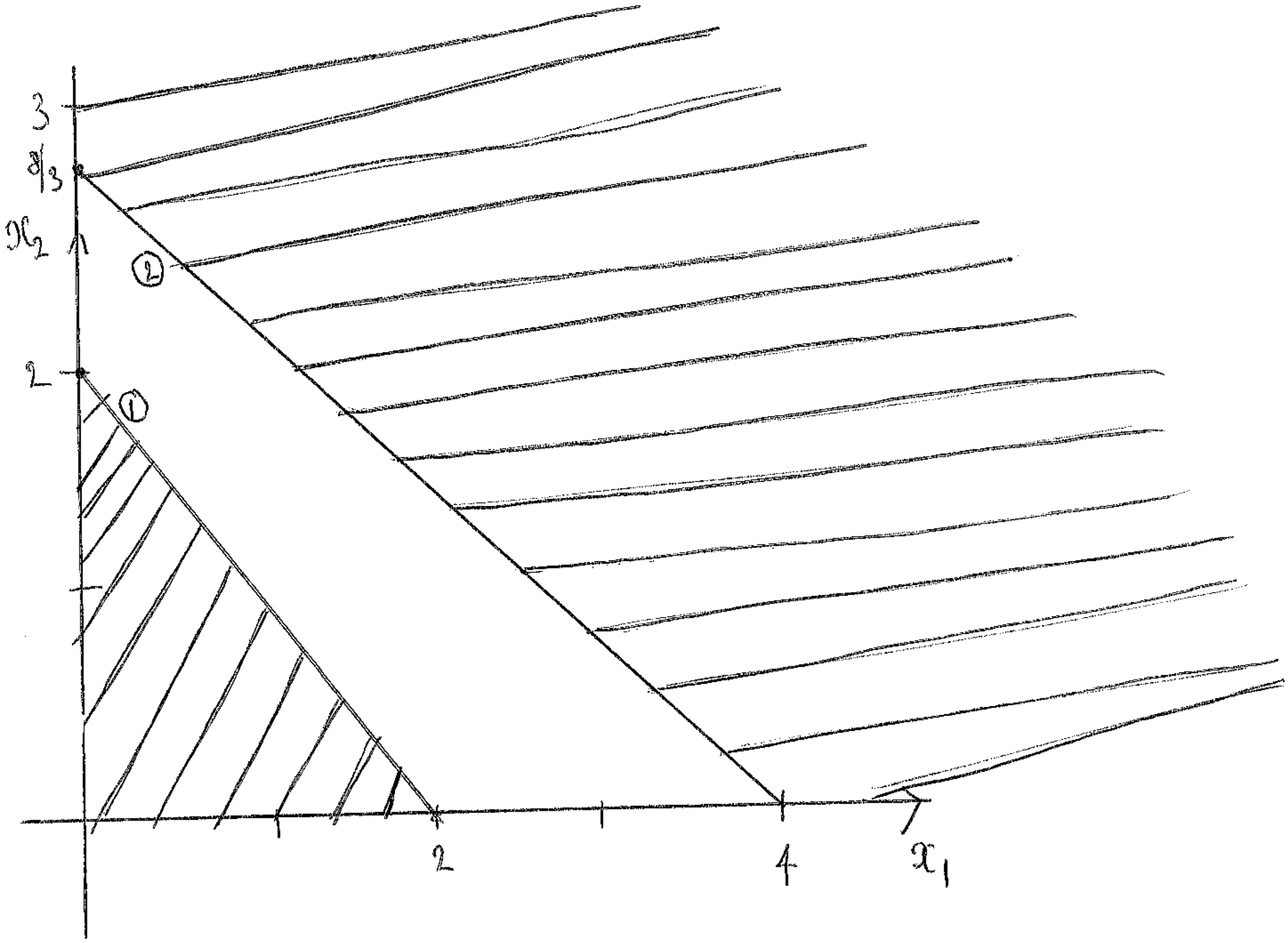
$(0, 2)$

$(2, 0)$

② $2x_1 + 3x_2 \doteq 8$

$(0, 8/3) = (0, 2\tfrac{2}{3})$

$(4, 0)$



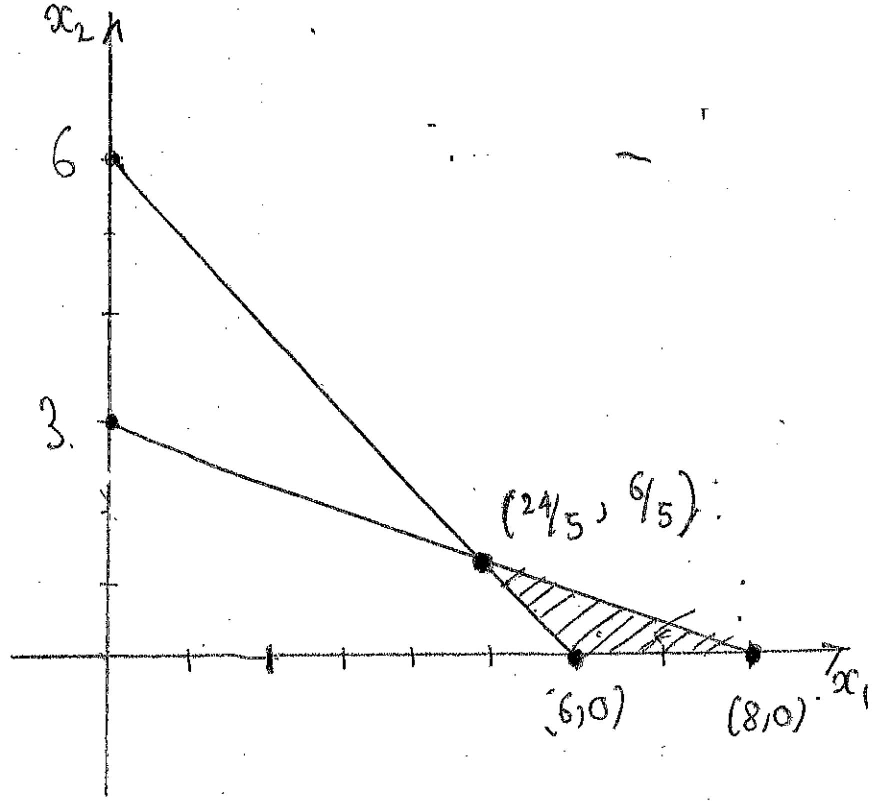
EMPTY F. REGION (NOTHING IS SHADED TWICE).

NO SOLN

7. Solve the following LP problems by the geometric method. Write down the solution and the value of the objective function there. Shade and label all vertices of the feasible region.

a)

$$\min C = 2x_1 + x_2$$
$$3x_1 + 8x_2 \leq 24$$
$$x_1 + x_2 \geq 6$$
$$x_1, x_2 \geq 0.$$



$3x_1 + 8x_2 = 24$

$(0, 3)$

$(8, 0)$

$x_1 + x_2 = 6$

$(0, 6)$

$(6, 0)$

$3x_1 + 8x_2 = 24$

$x_1 + x_2 = 6$

$-3x_1 - 3x_2 = -18$

$5x_2 = 6$

$x_2 = 6/5,\ x_1 = 6 - x_2 = 6 - \frac{6}{5} = 24/5$

VERTEX	VALUE OF C
$(24/5, 6/5)$	$54/5 = 10\,4/5$
$(6, 0)$	12
$(8, 0)$	16

MIN @ $(24/5, 6/5)$

$C = 10\,4/5$

b) max $P = 3x_1 + x_2$ Same constraints.

VERTEX	VAL. P
$(24/5, 6/5)$	$15\,3/5$
$(6, 0)$	18
$(8, 0)$	24

SOLN

MAX @ $(8, 0)$ $P = 24$

(8) Solve the following LP problems by the geometric method. Write down the solution and the value of the objective function there. Shade and label all vertices of the feasible region.

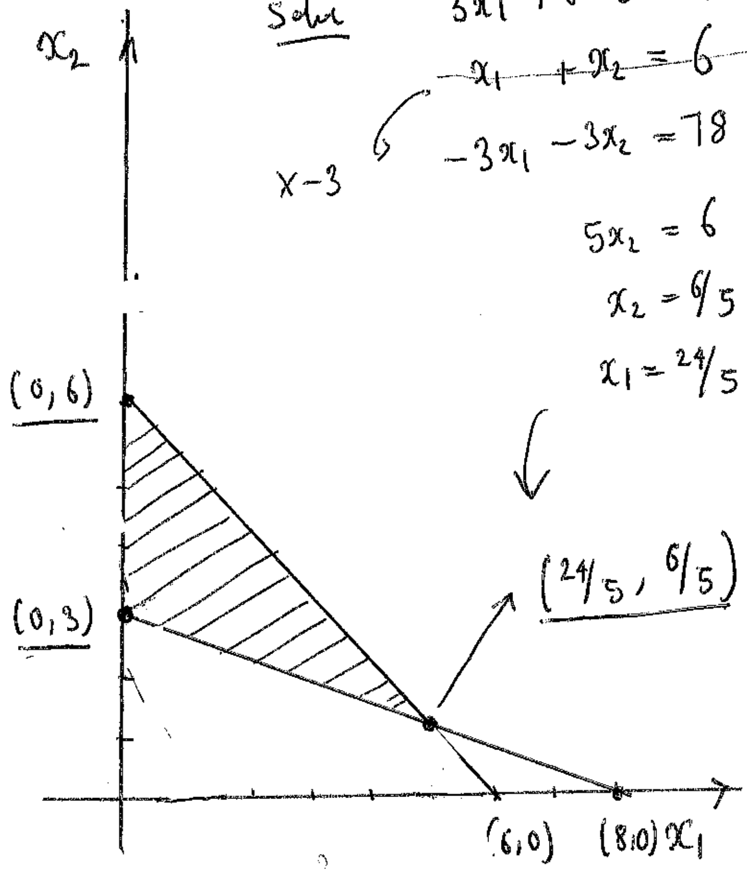
a)

$$\begin{aligned} \min C = 2x_1 &+ x_2 \\ 3x_1 + 8x_2 &\geq 24 \\ x_1 + x_2 &\leq 6 \\ x_1, x_2 &\geq 0. \end{aligned}$$

obj $2x_1 + x_2 = 2$

$(0, 2)$

$(1, 0)$



Solve

$3x_1 + 8x_2 = 24$

$\cancel{x_1 + x_2 = 6}$ $\times -3$

$-3x_1 - 3x_2 = -18$

$5x_2 = 6$

$x_2 = 6/5$

$x_1 = 24/5$

$(24/5, 6/5)$

$3x_1 + 8x_2 = 24$

$(0, 3)$

$(8, 0)$

$x_1 + x_2 = 6$

$(0, 6)$

$(6, 0)$

Vertices	C
$(0,6)$	6
$(0,3)$	3
$(24/5, 6/5)$	$10\,4/5$

min C @ $(0,3)$

$C = 3$.

b) max $P = 3x_1 + x_2$. Same constraints.

Vertex	P value
$(0,6)$	6
$(0,3)$	3
$(24/5, 6/5)$	$78/5$ ~ $15\,3/5$

max P @ $(24/5, 6/5)$

$P = 78/5$ ~ $15\,3/5$